Learning goals of the week:

- how to represent your data with plots and histograms
- understand what is a statistical vs. systematic uncertainty
- how to report numerically a measurement

Week 1

Data representation

Data graphical representation

The importance of a graphical representation of the data is obvious…

Fall time from 10 different heights with uncertainties

height (m)	time (s)	uncertainty (s)
0.498	0.33	± 0.05
0.676	0.28	± 0.05
0.805	0.44	± 0.05
0.970	0.49	± 0.05
1.12	0.45	± 0.05
1.28	0.52	± 0.05
1.43	0.64	± 0.05
1.59	0.60	± 0.05
1.72	0.59	± 0.05
1.89	0.55	± 0.05

100 measurements gaussian distributed
mean = 0, std deviation = 1

-0.69; -0.77; -0.037; -0.047; -0.88; 0.5; -1.7; -0.89; 1.4; 0.47; -0.081; 1.7; 0.27; -0.77; -0.19; -0.47; 1.1; 0.86; -1.4; 1; -0.78; 0.36; -0.08; -0.62; -0.31; -0.63; 0.33; -1.1; -1.3; 1.3; 1.2; 1.2; -0.45; 0.058; -1.2; 0.73; -0.3; 1.2; -0.48; -0.27; -0.25; 0.077; 1.8; 2.4; 0.51; 1.3; 2.1; -0.72; 1.1; 0.83; 0.055; -1.2; -3.8; -0.95; -0.25; 0.11; -0.38; 0.9; 0.16; 0.38; 2; -0.34; 0.16; -0.41; -1.8; 0.27; -1.3; -0.33; -0.33; -0.36; 1.7; -0.52; 0.84; 0.97; 1.8; 1.3; 1.1; -0.21; -1.1; -0.039; -0.33; -0.2; 0.81; -1.5; 0.73; 0.37; -0.39; 0.45; -0.44; -1.6; 2; -0.44; -0.19; -0.57; -0.094; 0.68; -0.19; 0.56; -0.37; -1.5

In the labs you will almost always encounter two kinds of graphical representations:

- graphs
- histograms

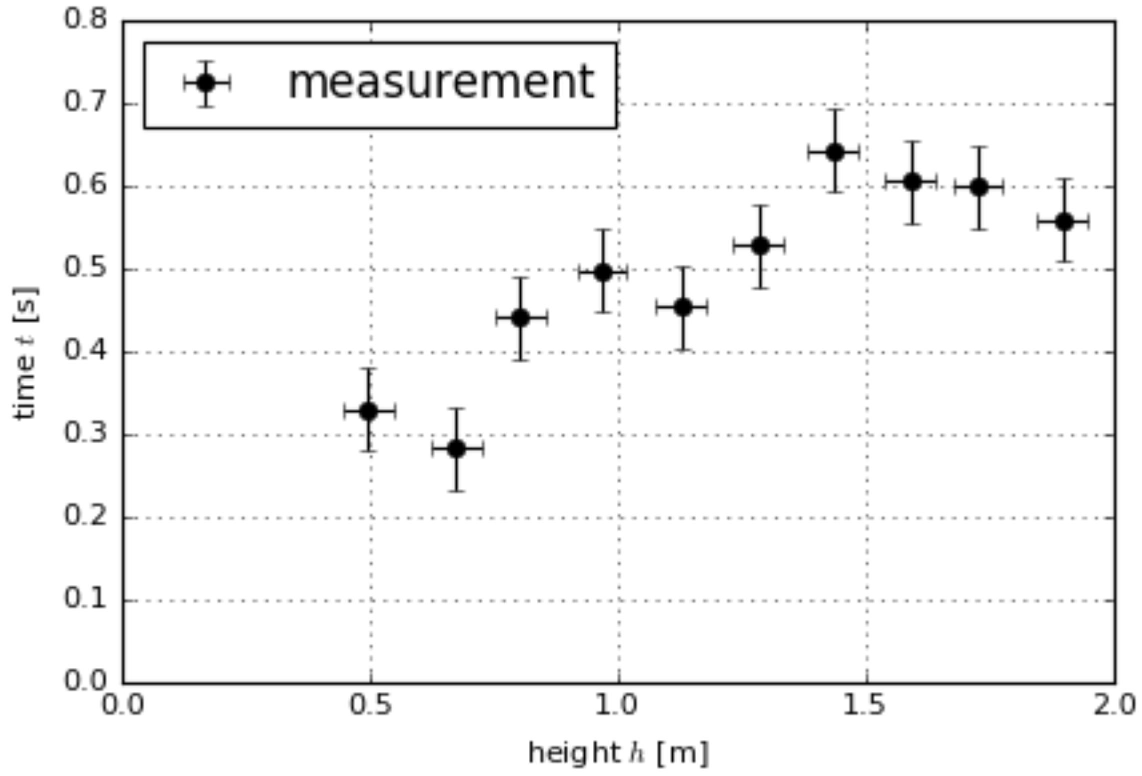
Scatter plots ≠ Histograms

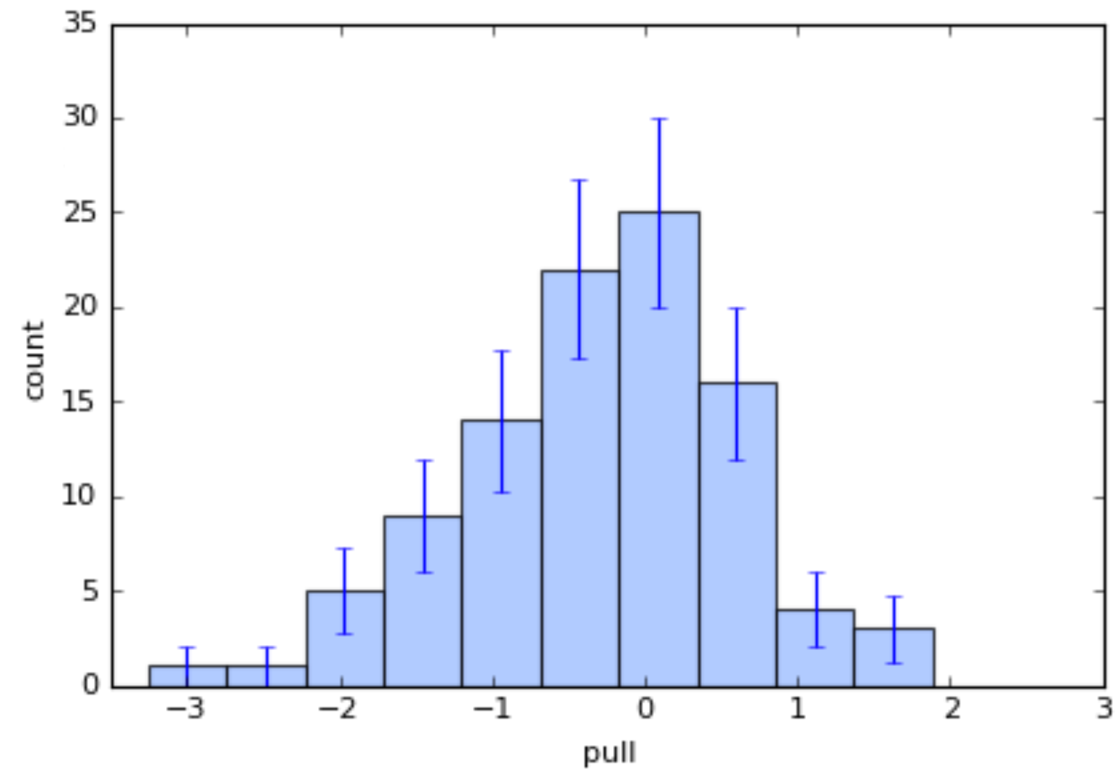
(2D) Scatter plots aka as graphs or simply plots represent data of the form x±δx,y±δy.

Histograms represent the frequency of a measurement

Fall time from 10 different heights with uncertainties

100 measurements gaussian distributed mean = 0, std deviation = 1





Scatter plots ≠ Histograms

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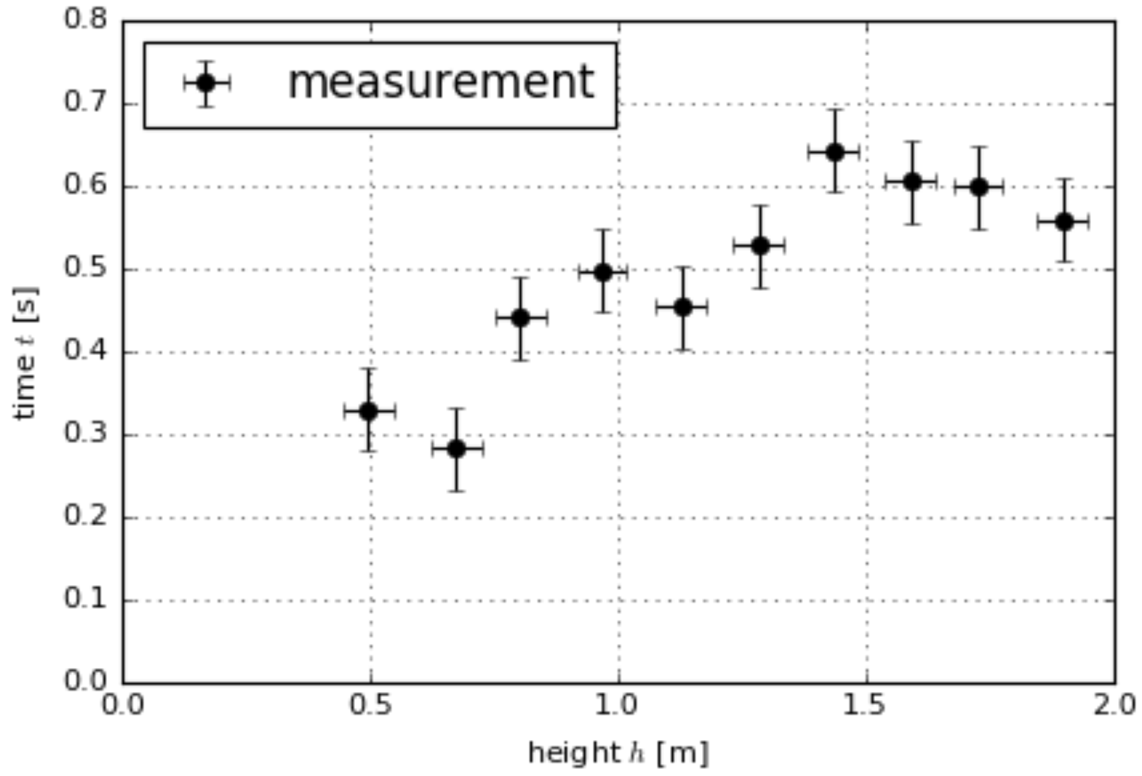
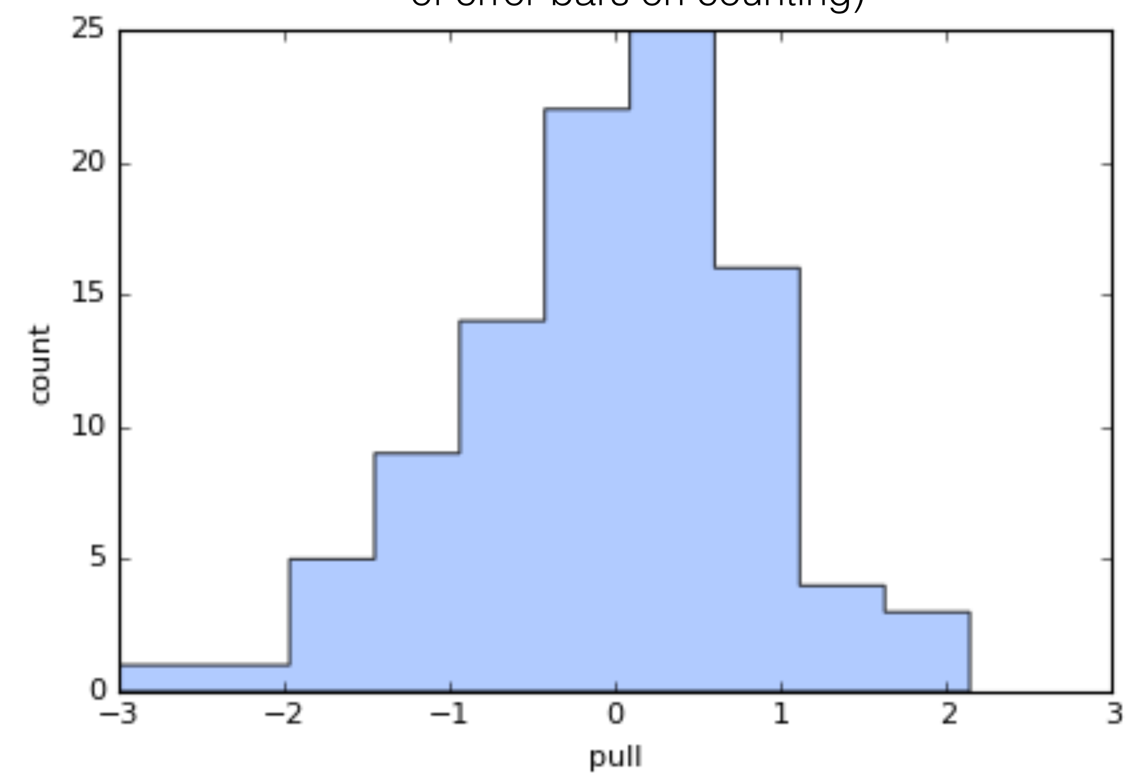
Fall time from 10 different heights with uncertainties



Histograms represent the frequency of a measurement

100 measurements gaussian distributed mean = 0, std deviation = 1

(we'll come back to the meaning of error bars on counting)



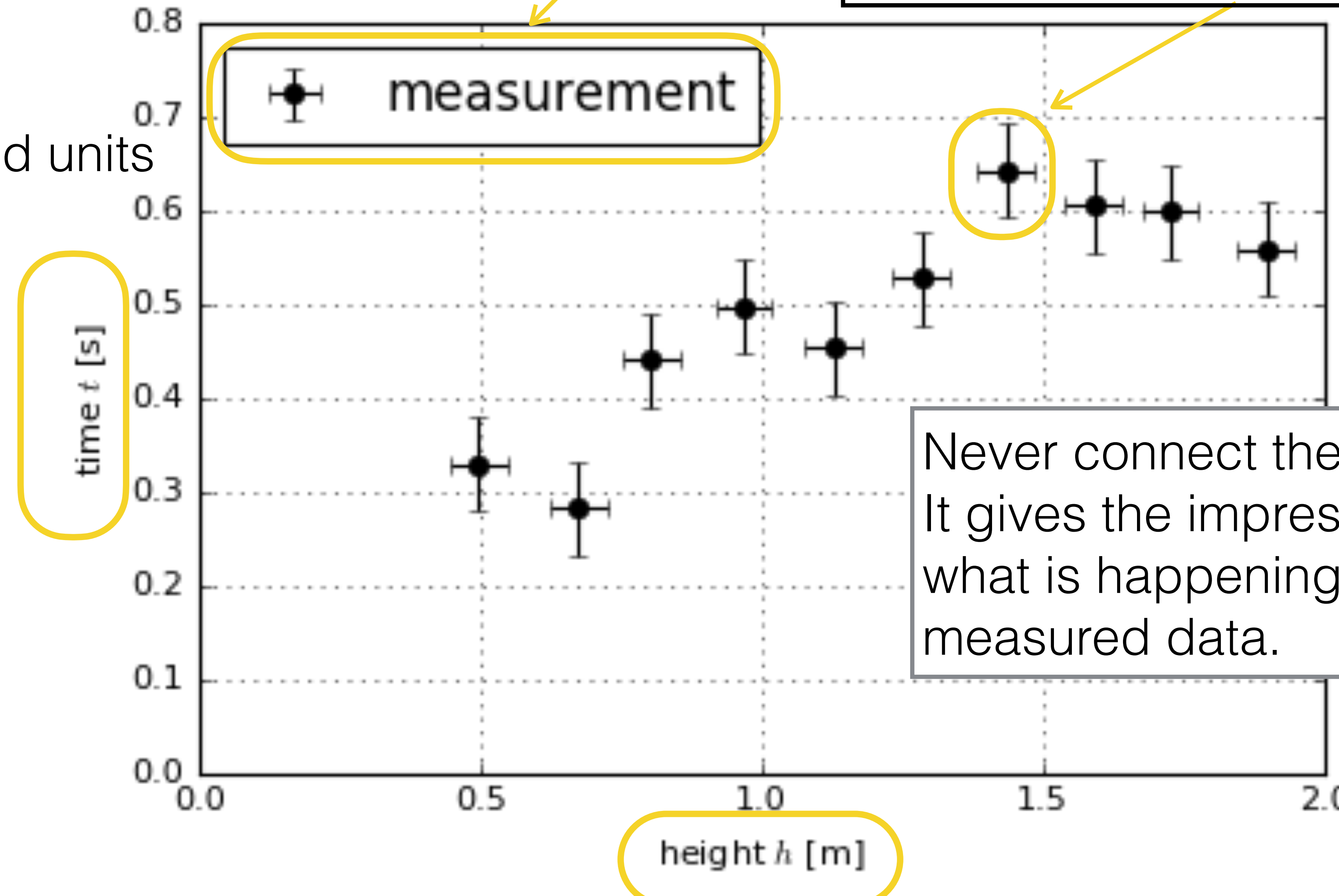
“Cosmetics”

In general a plot have to have readable axis, labels, legends, uncertainties

Legend

Data points have uncertainties in both x and y.If the error bars are smaller than the marker, *state it clearly in the caption.*

Y-axis label and units



Never connect the dots with a line ! It gives the impression you know what is happening in between the measured data.

X-axis label and units

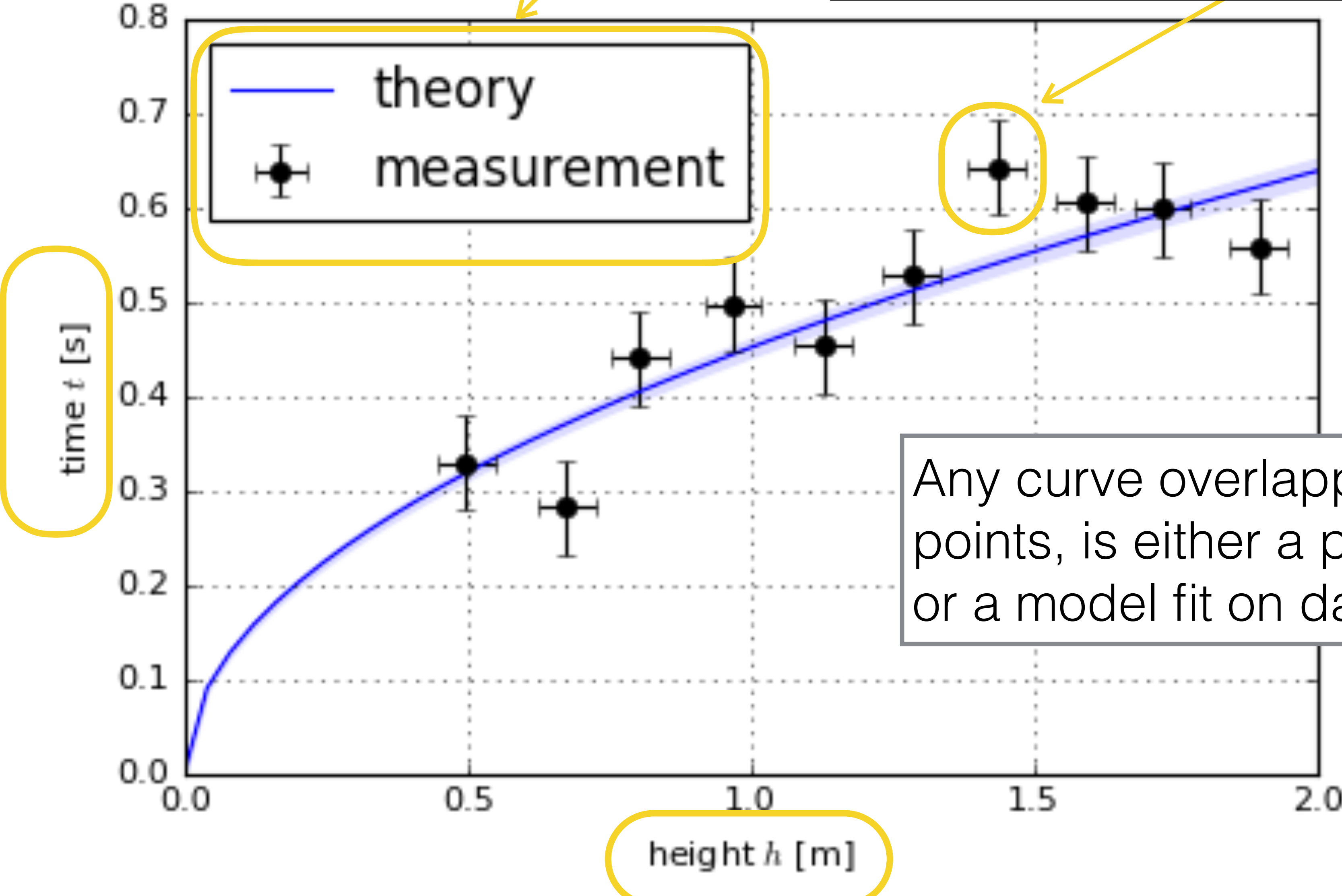
“Cosmetics”

In general a plot have to have readable axis, labels, legends, uncertainties

Legend

Data points have uncertainties in both x and y.If the error bars are smaller than the marker, *state it clearly in the caption.*

Y-axis label

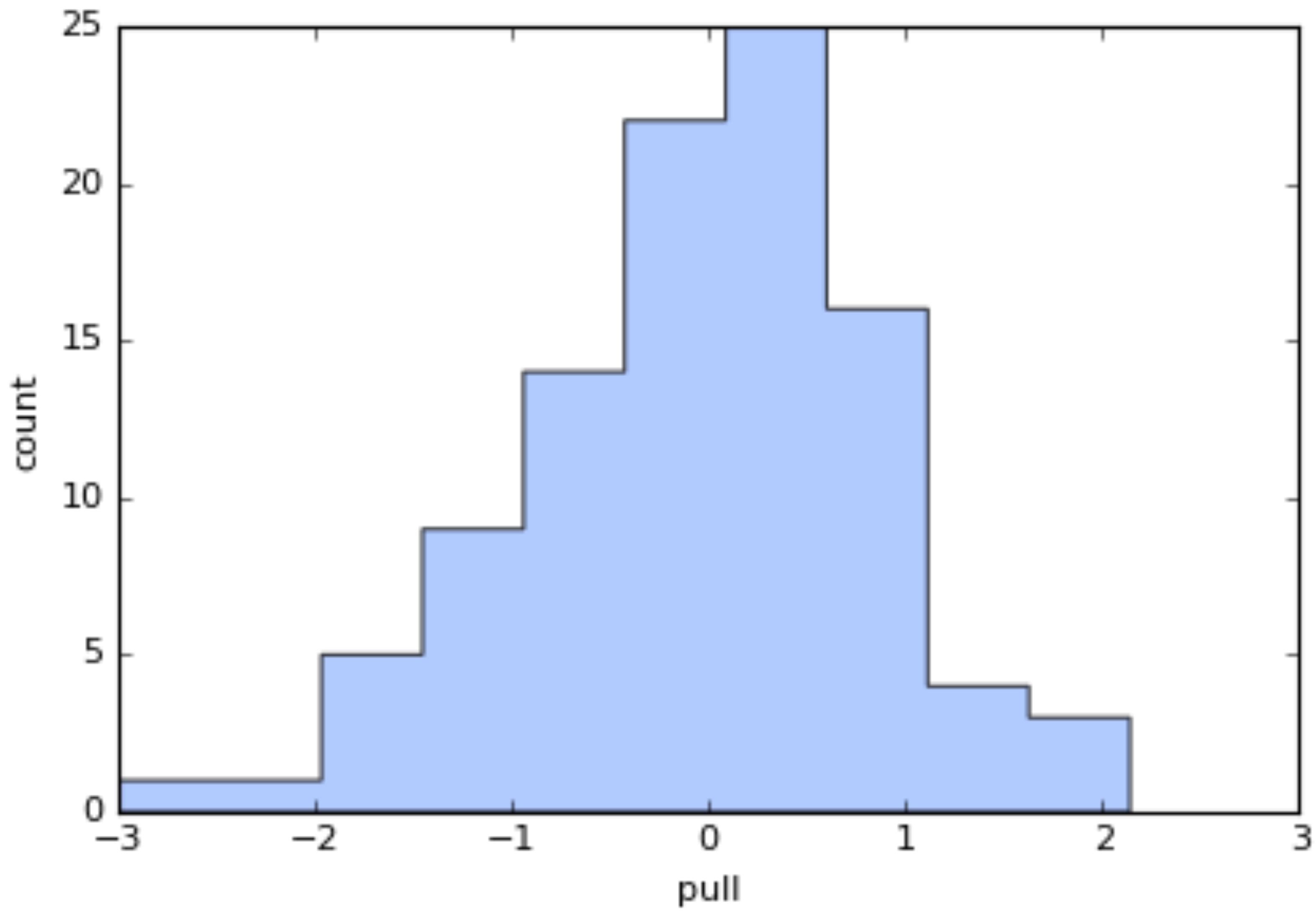


Any curve overlapped to data points, is either a prediction or a model fit on data

X-axis label

Histograms

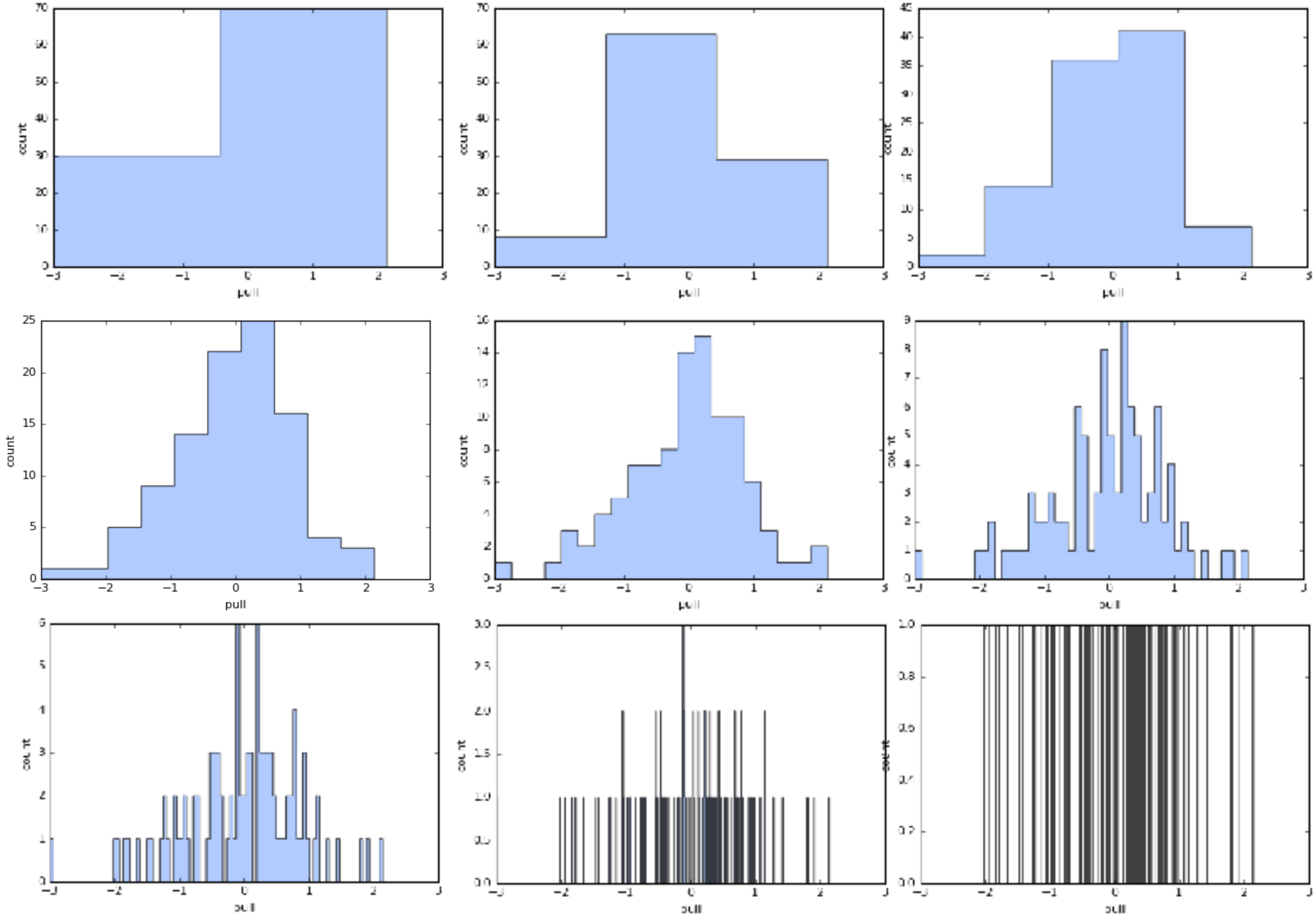
Classify the data in intervals “bins”: bin-width = (xmax- xmin) / nbins
Data are now represented by a vector of nbins: [1, 1, 5, 9, 14, 22, 25, 16, 4, 3]



Histograms binning choice

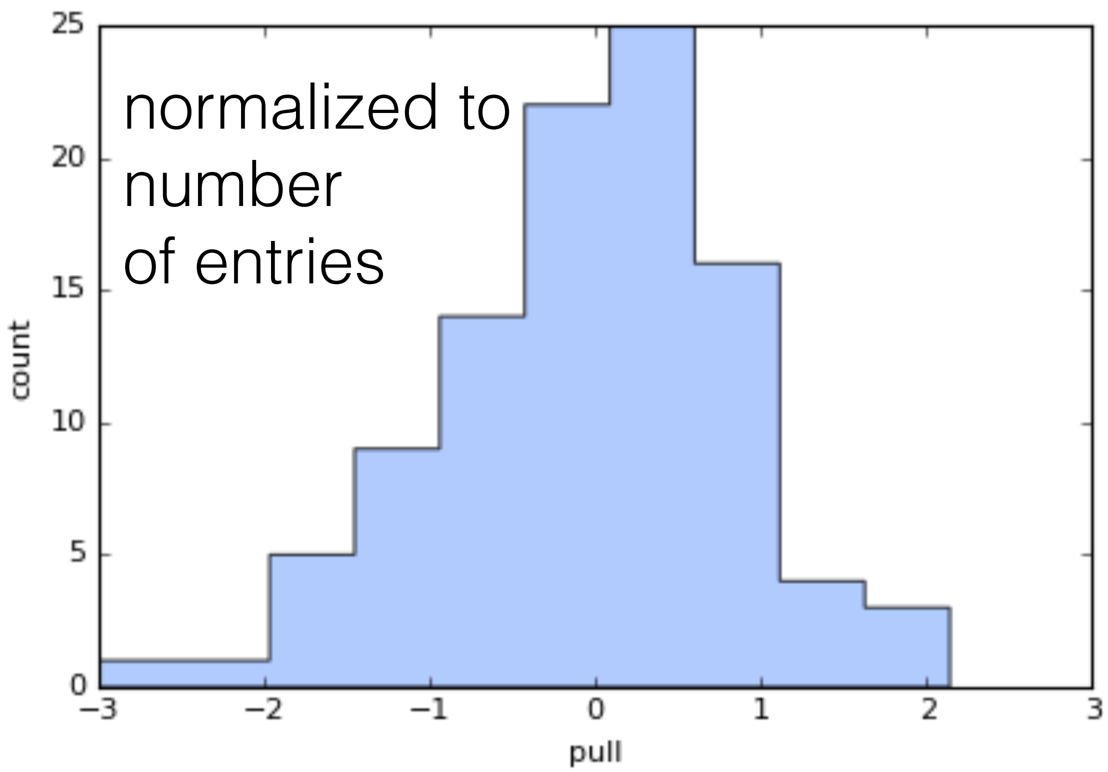
By binning your data, you generally lose information !

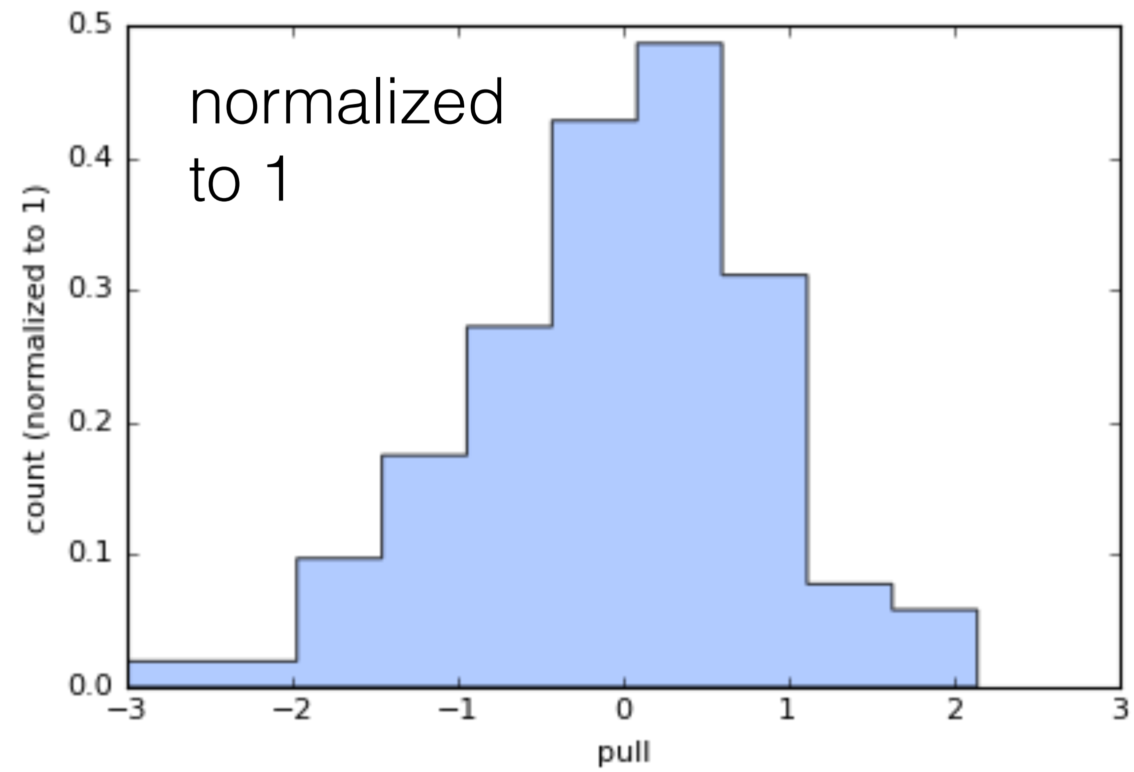
The choice of binning is important not to wash away structures



Histograms normalization

The area of the vertical bars is proportional to the number of entries in that bin.
The area of the histogram (the sum of the areas of the vertical bars) is proportional to the total number of entries: normalization

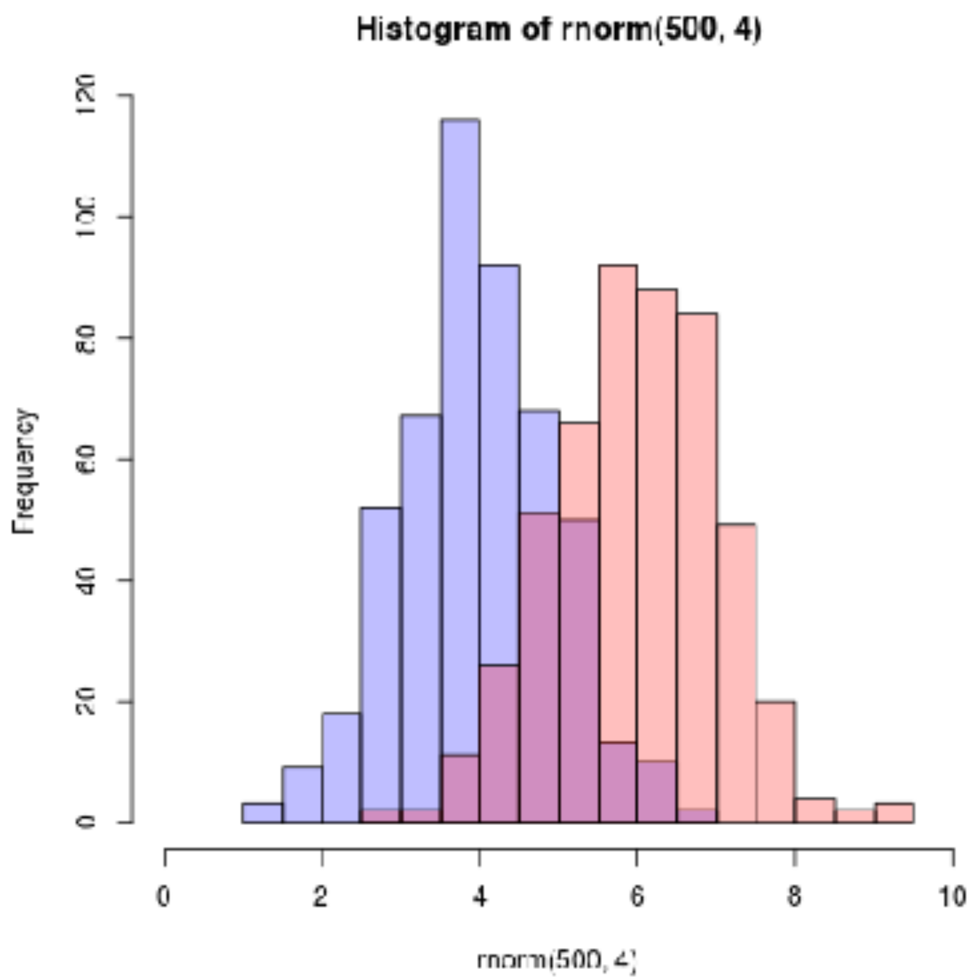


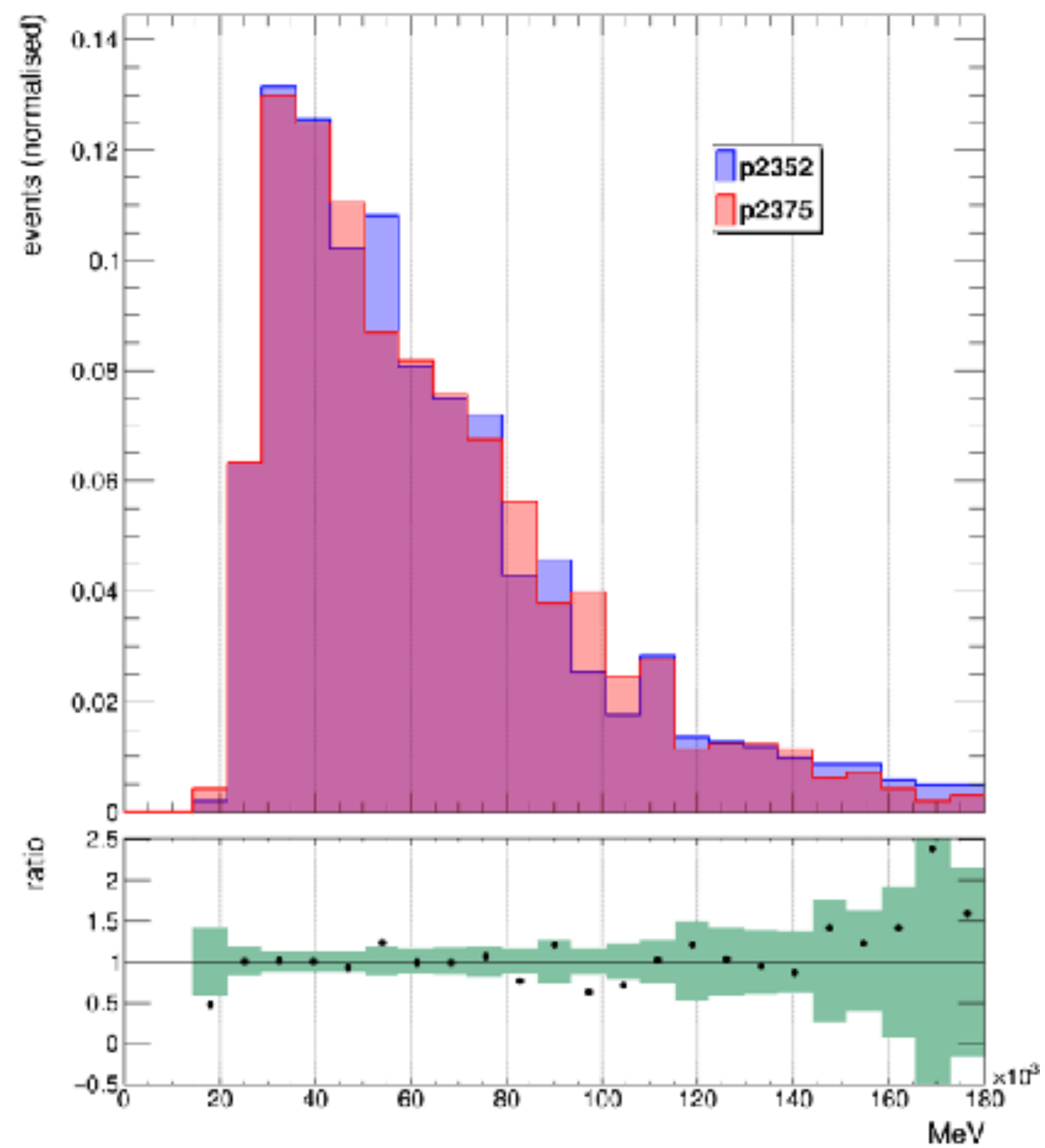


Overlapping histograms normalized to the same area (typically 1) allows to compare shapes ignoring an overall constant factor

Overlay histograms

Compare the shapes of normalized histograms:

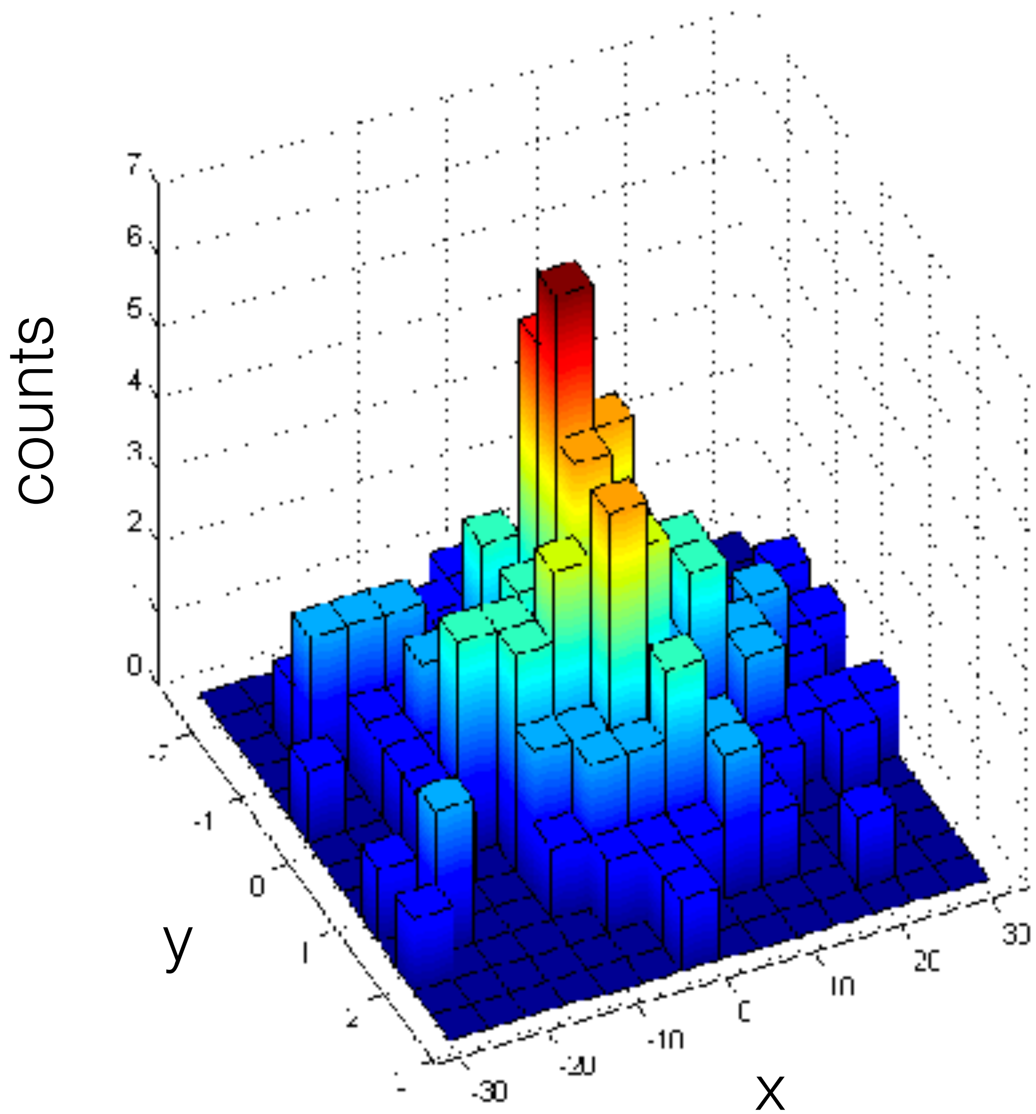




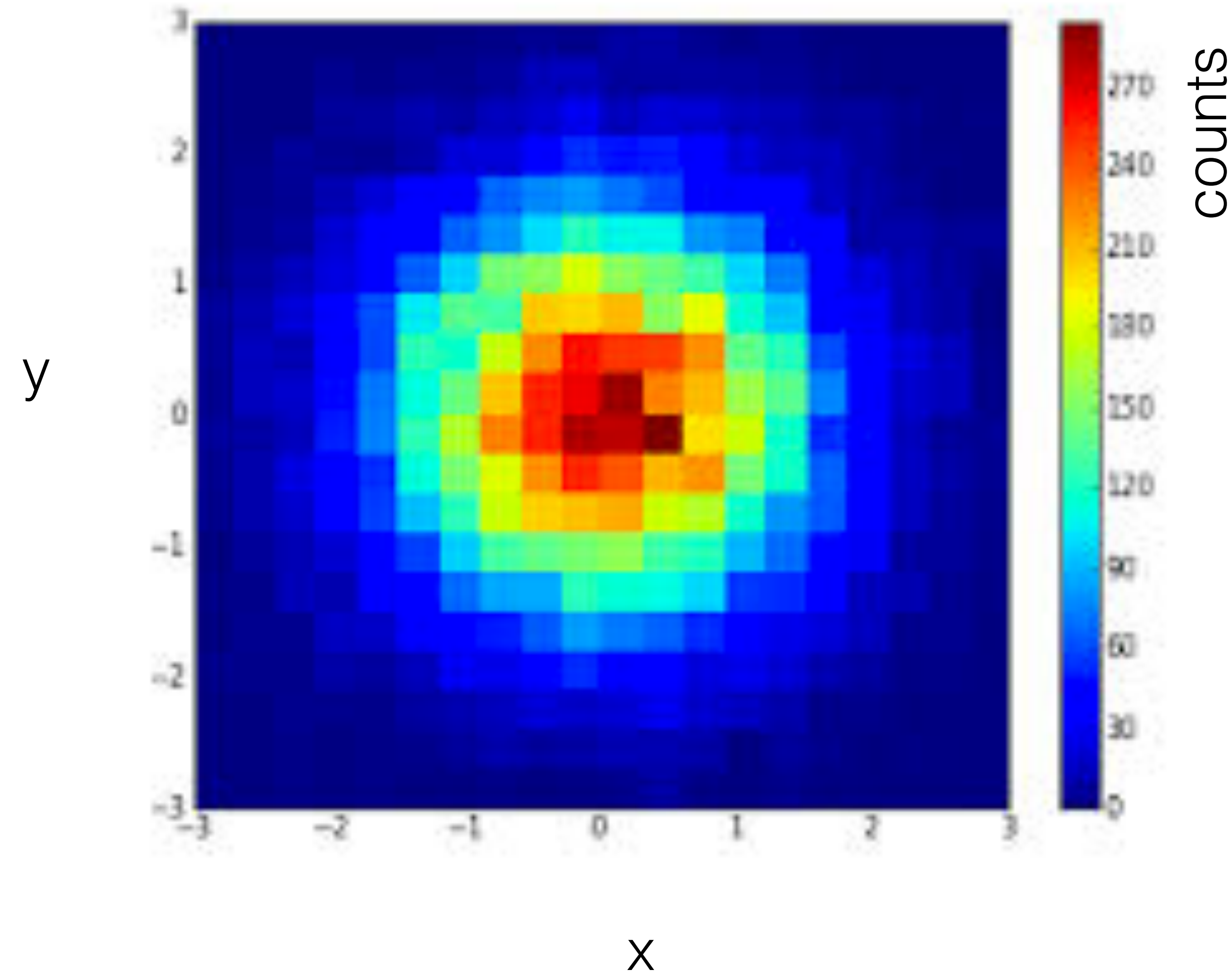
Use ratio plots to better show dis/agreements

2D Histograms

Bins can be defined in 2D i.e. (x_{min}, x_{max}) x (y_{min}, y_{max})



"Lego plot"

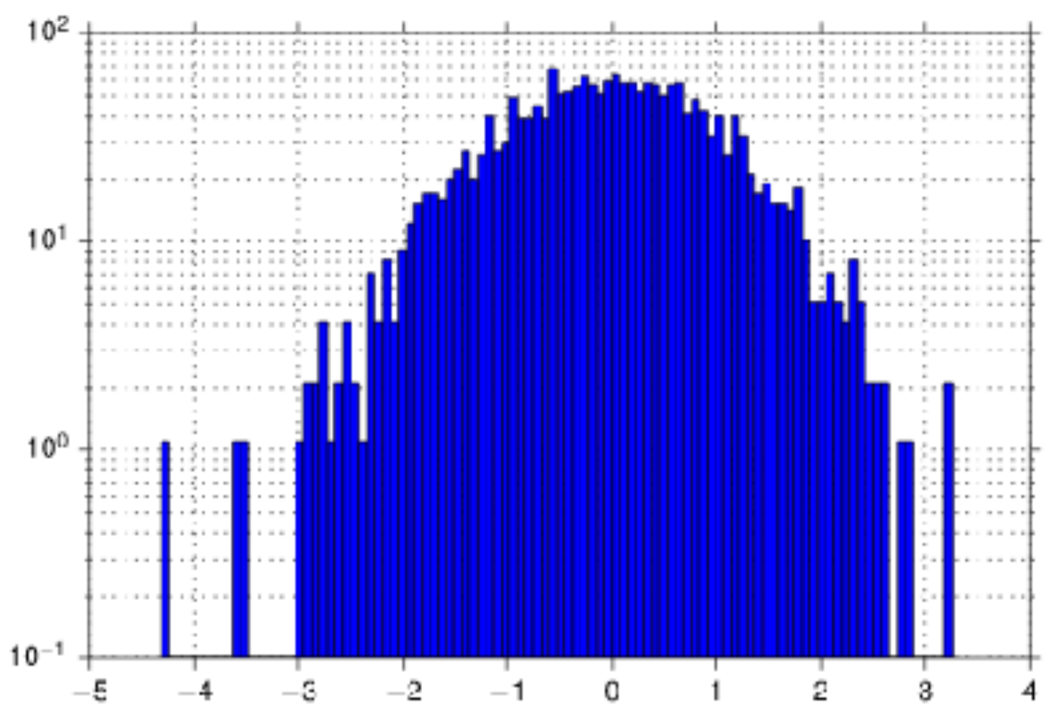


“xy plot”

Log scales

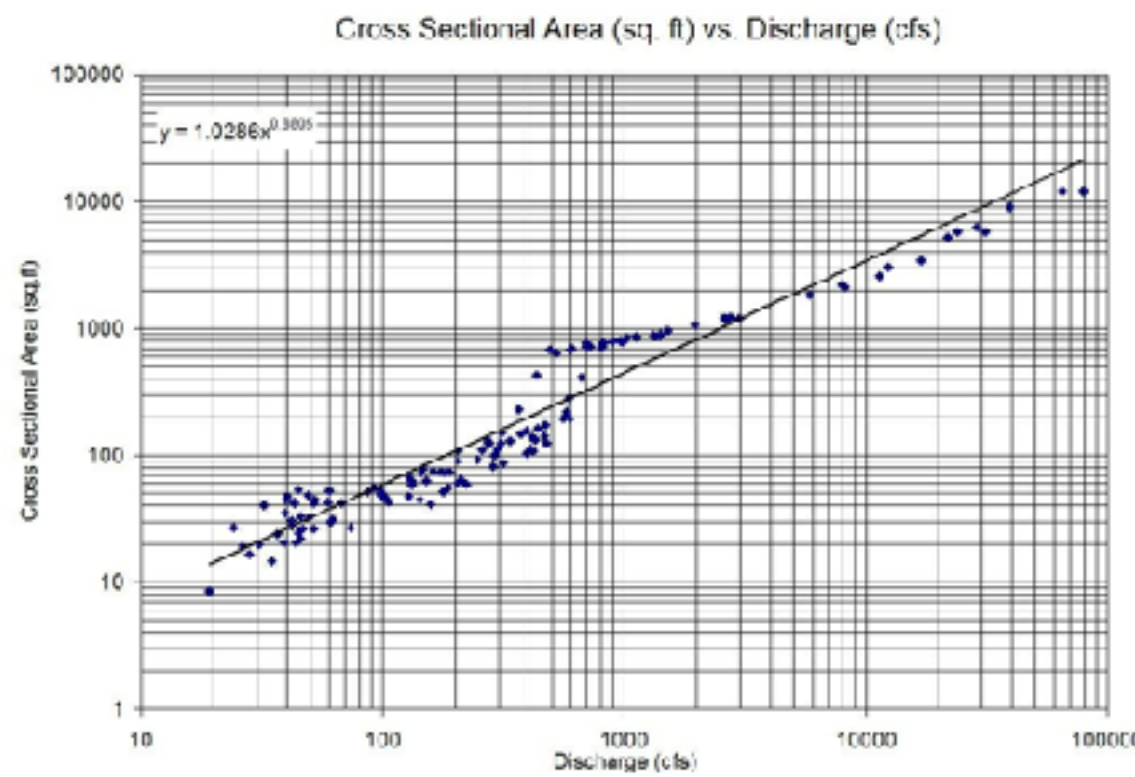
When you want to show data varying over several orders of magnitude, use log scales. (i.e. plot the log of the values on the axis) Both for graphs and histograms

counts



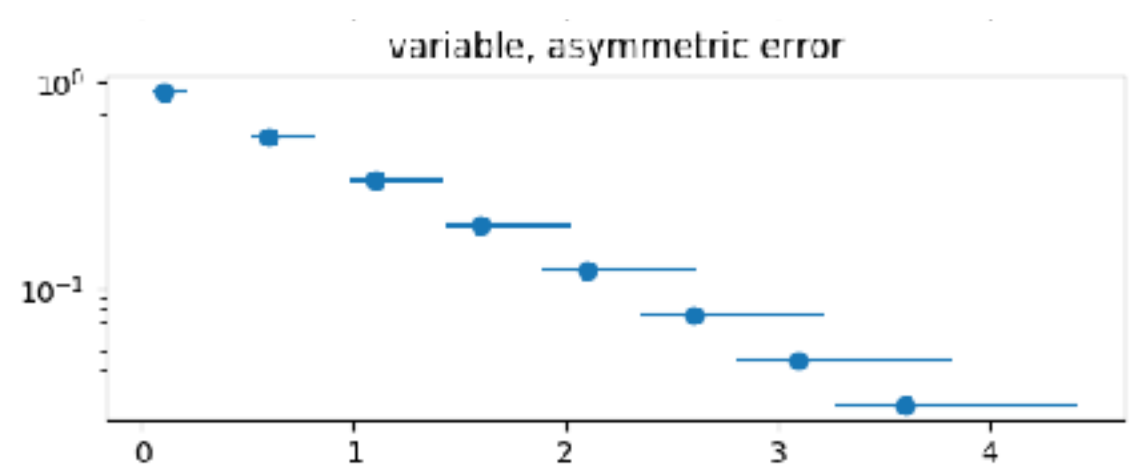
t-t_0 [msec]

“log” histogram



“log-log” graph

This is a better way to show the barycentre of the bin



Measurements uncertainties

Measurement Uncertainties

Every time you repeat a measurement you will get a slightly different result. The reason for this is that your measurement is affected by several un/controllable effects.

Scientific knowledge is based on experimental data i.e. measurements
⇒ **estimating the uncertainty of a measurement is of FUNDAMENTAL IMPORTANCE !**

Remember when writing your report.

There is NO big/small, high/low, long/short, fast/slow, good/bad…

Either you write a binary yes/no statement or you get a number with uncertainties

Measurement Uncertainties

Example: time with a stopwatch a running competition.
You have several uncertainties to take into account:

- your reaction time in starting / stopping the watch
- how far you are from the start / finish line —> do you hear the sound of the starter gun
- etc..

Some uncertainties can be reduced by repeating the measurements several time and taking averages, spread, etc.… (see later). These are called “statistical uncertainties”.

NB: sometimes you cannot repeat the measurement several times (as in the example above)

Other sources of uncertainties cannot be reduced by repeating the measurements several times (e.g. clock running slow/fast, sound propagation time from the starter to your position, etc..). These are called “systematic uncertainties”.

Sometimes the dividing line between statistical/systematic uncertainty is blurred !

Systematic uncertainties

Systematic effects ≠ Systematics uncertainties ≠ Systematic mistakes

Systematic effects = instrument bias, backgrounds (e.g.noise subtractions), etc..
(e.g. stopwatch running slow)

Systematic uncertainties = the uncertainty in estimating a systematic effect
(e.g. the uncertainty coming from the calibration of the stopwatch)

Systematic mistake = result of neglecting such effects

The systematic uncertainty is a statement made by the experimenters about their understanding of their own equipment

In general syst. unc. will not decrease if you repeat the measurement several times. Some systematic uncertainties do get reduced repeating the measurements, e.g. syst unc. associated to a calibration: the larger the calibration sample the smaller will be the uncertainty.

When syst unc. > stat unc. the precision of the result will not be improved by taking more data; it will only improve by better understanding the experimental setup, or by building a better setup.

Systematic uncertainties

Known unknowns vs. unknown unknowns

Unknown (unsuspected) sources: these are very delicate.
Your gut feeling tells you that there might be an effect on your results coming from a particular setup or assumption. The first step is to perform a "cross-check".
E.g.

- vary the range of data used for extraction of the result
- use only a subset of the data (e.g. split the data into two categories)
- change binning of the histogram
- change the parameterization or the fit technique

If the analysis passes the "cross-check" then don't do anything. (don't add all sorts of small discrepancies to the systematic uncertainty !)

If the analysis doesn't pass the "cross-check":

- check the test, might be a mistake there
- check the analysis, might be a mistake there
- the effect is real:
    - the check is promoted to systematic effect, correct for it
    - can't correct for it add it to the systematic uncertainty

Systematic uncertainties

Suggestion:

As soon as you are done reading the manual of the experiment, try to figure out what systematic sources of uncertainty affect your measurement.

This will save you a lot of time when trying to understand your measurements and debug your setup !

Quick reminder: How to report measurements

How to report a measurement

Presenting a measurement with only one number

"I measured x"

without quoting its uncertainty is not wrong. It's meaningless !

You will ALWAYS have to add your best evaluation of the uncertainty to your results

$$x \pm \delta x$$

We will come back to the "meaning of δx" later, when going to the details it can be tricky…

E.g. Are these measurements compatible ?

$x = 1$ $y = 5$

$x = 1 \pm 0.1$ $y = 5 \pm 0.01$ ← **absolute unc. = δx** = 0.01
relative unc. = δx /x = 0.01/5 (= 0.002 = 2 per mille)

$x = 1 \pm 0.1$ $y = 5 \pm 3$

$x = 1 \pm 0.1$ $y = 5 \pm 10$

Rounding reminder

General naive rule:

If the last-but-one digit is followed by 5, 6, 7, 8, or 9, round the number up.

if the last-but-one digit is followed by 0, 1, 2, 3, or 4

Rounding - pedantic rule

PDG recommendation: (Particle Data Group: pdg.lbl.gov)

if the three highest order digits of the uncertainty lie between 100 and 354, we round to two significant digits.

if they lie between 355 and 949, we round to one significant digit

if they lie between 950 and 999, we round up to 1000 and keep two significant digits.

In all cases, the central value is given with a precision that matches that of the uncertainty.

Example:

0.827 ± 0.050 would appear as 0.827 ± 0.050

0.827 ± 0.119 would appear as 0.83 ± 0.12

0.827 ± 0.367 would turn into 0.8 ± 0.4.

Significant digits

Rule #1: The last significant figure in any stated answer should be of the same order of magnitude (in the same decimal position) as the uncertainty.

$$x = 1.2 \pm 0.1$$
$$x = 5 \pm 3$$

Rule #2: To reduce inaccuracies caused by rounding, any number to be used in subsequent calculations should normally retain at least one significant figure more than is finally justified. The final result should be rounded to remove to remove these insignificant digits

Rule #3: Experimental uncertainties should almost always be rounded to one significant figure. (Only for very high precision measurements state two digits)

Exception to rule #3: If the leading digit in the uncertainty is a 1, then keeping two significant figures in the uncertainty may be better.
E.g. $\delta x = 0.14$. Rounding to $\delta x = 0.1$ would be a reduction of 40% in the uncertainty —> better keeping $\delta x = 0.14$
(one can also argue 0.24 —> 0.2 is a 20% reduction)

Precision and Accuracy

Precision:

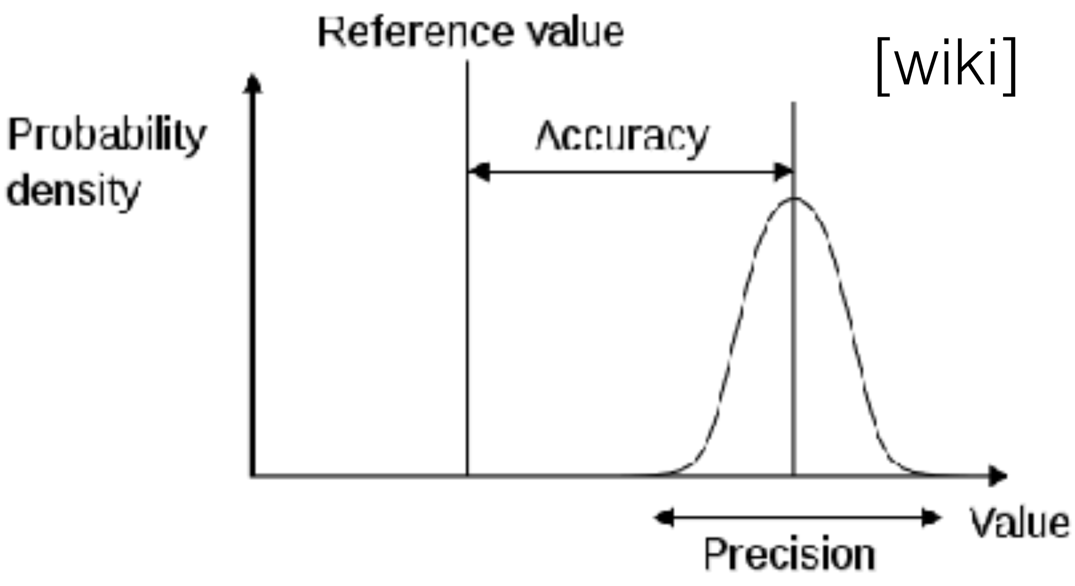
how reproducible is the measurement under identical conditions.
Or how well an instrument reproduces a certain output signal given a constant input

Accuracy:

how close the measured value is to the “reference”, “nominal” value;
or how well the behaviour of the measuring instrument agrees with its design behaviour.

One can be very precise, but not accurate (always measuring exactly the same, but wrong value …) and vice-versa.

More measurements may increase the precision, but not the accuracy



[wiki]

Some more definitions

(Intrinsic) Resolution is the smallest change in a measured value that the instrument can detect.

Measurement range is difference between largest and smallest input value that an instrument is capable of measuring/reading

Dynamic range is the ratio between measurement range and the resolution (quoted usually as log value “decibel=dB” in base-10)

Bandwidth is the difference between the upper and lower frequencies (in electronics) that an instrument is capable of measuring. Or the maximal throughput for data transfer (computing)

Some more definitions

Calibration is the process of comparing the response of a device with unknown accuracy to a reference device with a known accuracy and precision.

"Operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties (of the calibrated instrument or secondary standard) and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication.”

BIPM:

Bureau International des Poids et Mesures

see http://www.bipm.org/

Bibliography

Plotting :
https://matplotlib.org/

Uncertainties:
Taylor: Chapters 1-5