# Verteilungsfunktionen und ihre charakteristischen Kenngrößen

## Die Binomialverteilung

Die Verteilungsfunktion in symbolischer Form:

```
PDF[BinomialDistribution[n, p], k]
```

$$\begin{cases} (1-p)^{-k+n} p^k \text{Binomial}[n, k] & 0 \le k \le n \\ 0 & \text{True} \end{cases}$$

Mit konkreten Parameterwerten:

```
PDF[BinomialDistribution[10, 0.4], k]
```

$$\begin{cases} 0.4^k\, 0.6^{10-k} \text{Binomial}[10, k] & 0 \le k \le 10 \\ 0 & \text{True} \end{cases}$$

Der Mittelwert symbolisch:

```
Mean[BinomialDistribution[n, p]]
```

$n\, p$

Der Mittelwert numerisch :

```
Mean[BinomialDistribution[10, 0.4]]
```

4.

Die Varianz symbolisch:

```
Variance[BinomialDistribution[n, p]]
```

$n\,(1-p)\,p$

Die Varianz numerisch:

```
Variance[BinomialDistribution[10, 0.4]]
```

2.4

Alternativ nennt man die Varianz auch das zweite zentrale Moment der Verteilung:

```
Simplify[CentralMoment[BinomialDistribution[n, p], 2]]
```

$-n\,(-1+p)\,p$

Die Standardabweichung symbolisch:

```
StandardDeviation[BinomialDistribution[n, p]]
```

$\sqrt{n\,(1-p)\,p}$

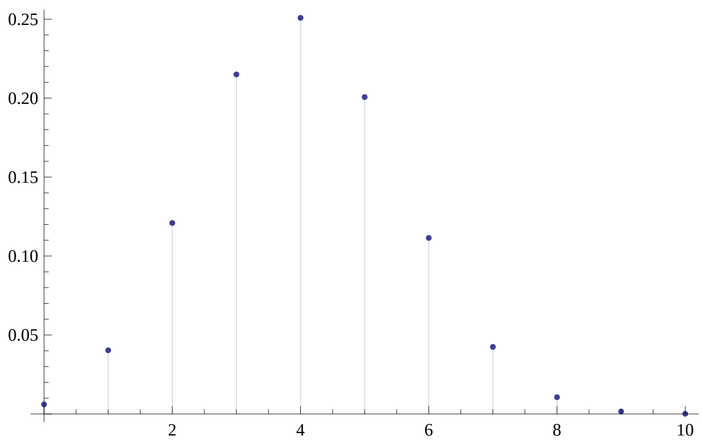
Die Standardabweichung numerisch

```
StandardDeviation[BinomialDistribution[10, 0.4]]
```

1.54919

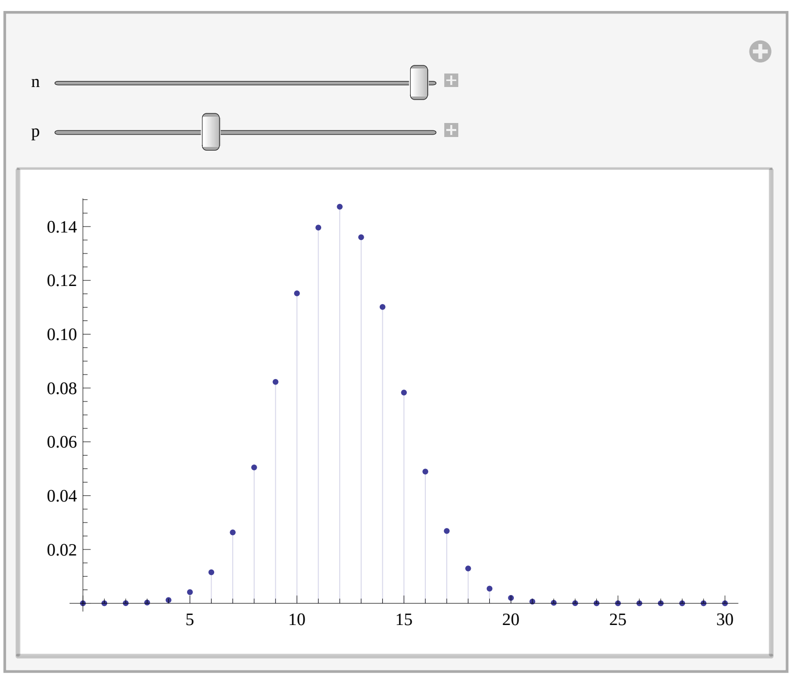
Ein Plot der Verteilungsfunktion mit bekannten Parametern:

```
DiscretePlot[PDF[BinomialDistribution[10, 0.4], k], {k, 0, 10}]
```




Parameterabhängigkeit interaktiv mit Manipulate darstellen:

```
Manipulate[DiscretePlot[PDF[BinomialDistribution[n, p], k], {k, 0, n}],
 {{n, 30}, 2, 30, 1}, {{p, 0.4}, 0, 1}]
```

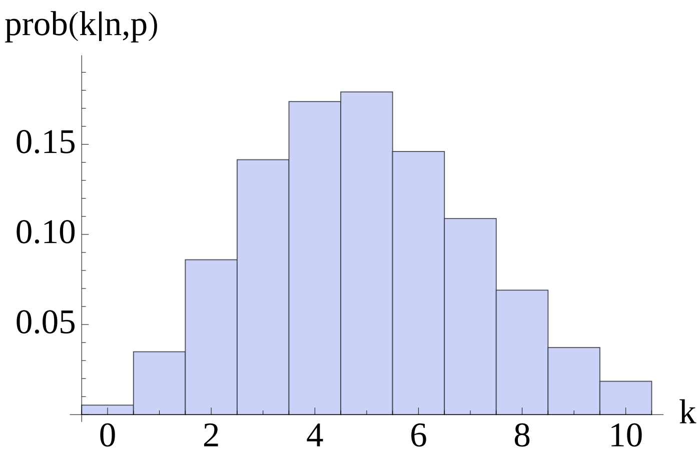



Zufallszahlen aus der Binomialverteilung ziehen und Histogram plotten:

In[71]:=
```
r = RandomVariate[BinomialDistribution[10, 0.4], 1000];
```

In[78]:=
```
Histogram[r, {-0.5, 10.5, 1}, "Probability",
 LabelStyle → 18, AxesLabel → {"k", "prob(k|n,p)"}]
```

Out[78]=




# Die Poissonverteilung

```
PDF[PoissonDistribution[μ], x]
```

$$\begin{cases} \frac{e^{-\mu}\mu^x}{x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

```
PDF[PoissonDistribution[5], x]
```

$$\begin{cases} \frac{5^x}{e^5\, x!} & x \geq 0 \\ 0 & \text{True} \end{cases}$$

```
Mean[PoissonDistribution[μ]]
```

$\mu$

```
Mean[PoissonDistribution[5]]
```

5

```
Variance[PoissonDistribution[μ]]
```

$\mu$

```
Variance[PoissonDistribution[5]]
```

5

```
CentralMoment[PoissonDistribution[μ], 2]
```

$\mu$

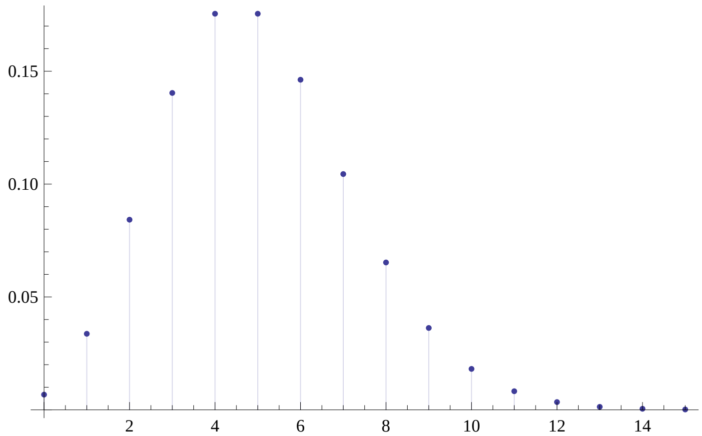
```
StandardDeviation[PoissonDistribution[μ]]
```

$\sqrt{\mu}$

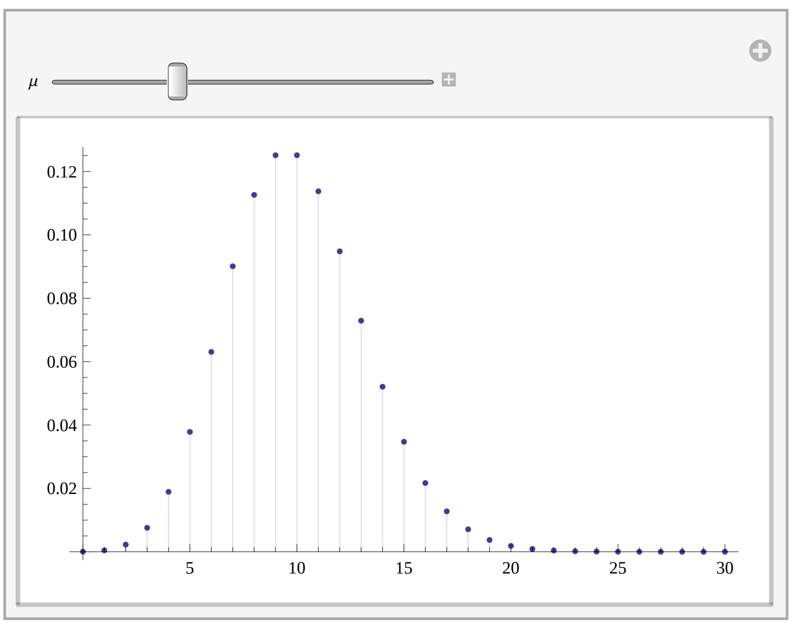
```
StandardDeviation[PoissonDistribution[5.0]]
```

2.23607

```
DiscretePlot[PDF[PoissonDistribution[5], k], {k, 0, 15}]
```




```
Manipulate[DiscretePlot[PDF[PoissonDistribution[μ], k],
  {k, 0, 30}, PlotRange → All], {{μ, 10}, 1, 30}]
```

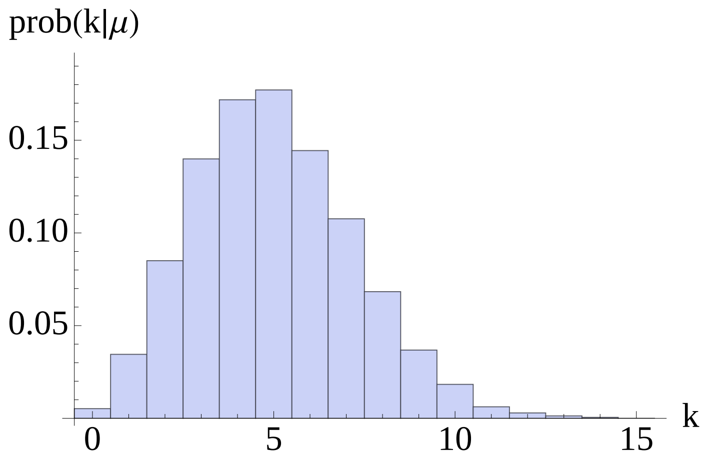



Zufallszahlen aus der Poissonverteilung ziehen und Histogram plotten:

```
In[74]:= r = RandomVariate[PoissonDistribution[5], 10 000];
```

In[77]:=
```
Histogram[r, {-0.5, 15.5, 1}, "Probability",
 LabelStyle → 18, AxesLabel → {"k", "prob(k|μ)"}]
```

Out[77]=




# Die Gammaverteilung

```
PDF[GammaDistribution[α, β], x]
```

$$\begin{cases} \frac{e^{-\frac{x}{\beta}} x^{-1+\alpha} \beta^{-\alpha}}{\text{Gamma}[\alpha]} & x > 0 \\ 0 & \text{True} \end{cases}$$

```
PDF[GammaDistribution[4, 1], x]
```

$$\begin{cases} \frac{1}{6} e^{-x} x^3 & x > 0 \\ 0 & \text{True} \end{cases}$$

```
Mean[GammaDistribution[α, β]]
```

$\alpha\,\beta$

```
Mean[GammaDistribution[4, 1]]
```

4

```
Variance[GammaDistribution[α, β]]
```

$\alpha\,\beta^2$

```
Variance[GammaDistribution[4, 1]]
```

4

```
CentralMoment[GammaDistribution[α, β], 2]
```

$\alpha\,\beta^2$

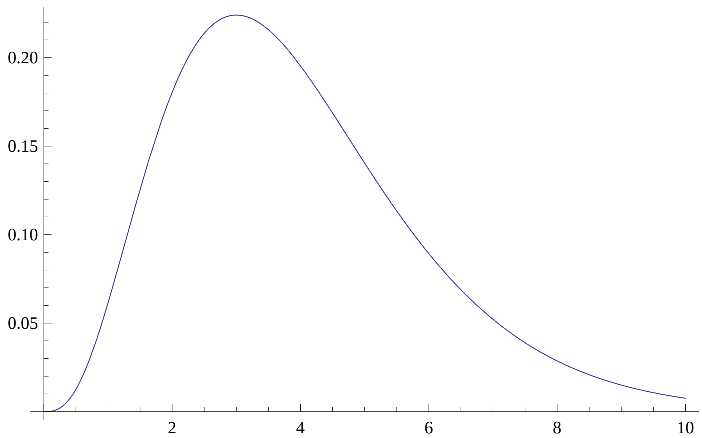
```
StandardDeviation[GammaDistribution[α, β]]
```

$\sqrt{\alpha}\;\beta$

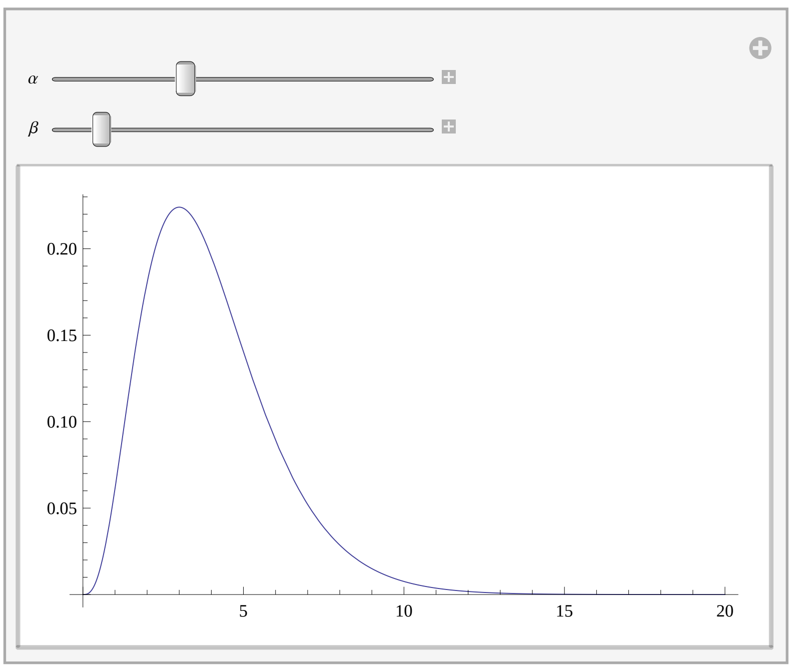
```
StandardDeviation[GammaDistribution[4, 1]]
```

2

```
Plot[PDF[GammaDistribution[4, 1], x], {x, 0, 10}]
```




```
Manipulate[Plot[PDF[GammaDistribution[α, β], x], {x, 0, 20}, PlotRange → All],
 {{α, 4}, 1, 10}, {{β, 1}, 0.1, 10, 0.1}]
```

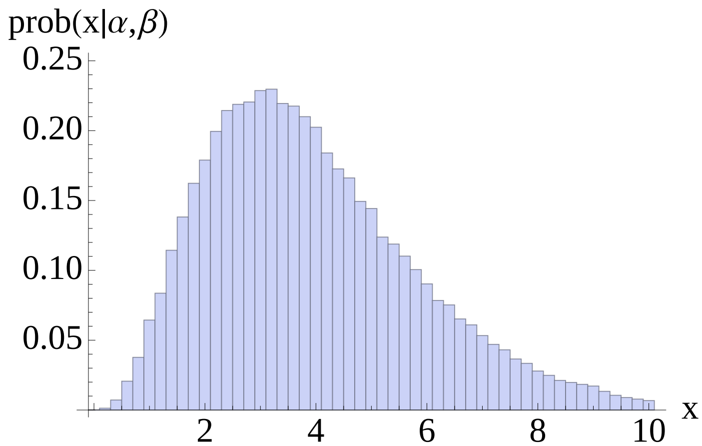



Zufallszahlen aus der Gammaverteilung ziehen und Histogram plotten:

```
In[98]:= r = RandomVariate[GammaDistribution[4, 1], 100 000];
```

In[99]:=
```
Histogram[r, {-0.1, 10.1, 0.2}, "ProbabilityDensity",
 LabelStyle → 18, AxesLabel → {"x", "prob(x|α,β)"}]
```

Out[99]=




# Die Normalverteilung

```
PDF[NormalDistribution[μ, σ], x]
```

$$\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma}$$

```
PDF[NormalDistribution[5, 2], x]
```

$$\frac{e^{-\frac{1}{8}(-5+x)^2}}{2\sqrt{2\pi}}$$

```
Mean[NormalDistribution[μ, σ]]
```

$\mu$

```
Mean[NormalDistribution[5, 2]]
```

5

```
Variance[NormalDistribution[μ, σ]]
```

$\sigma^2$

```
Variance[NormalDistribution[5, 2]]
```

4

```
CentralMoment[NormalDistribution[μ, σ], 2]
```

$\sigma^2$

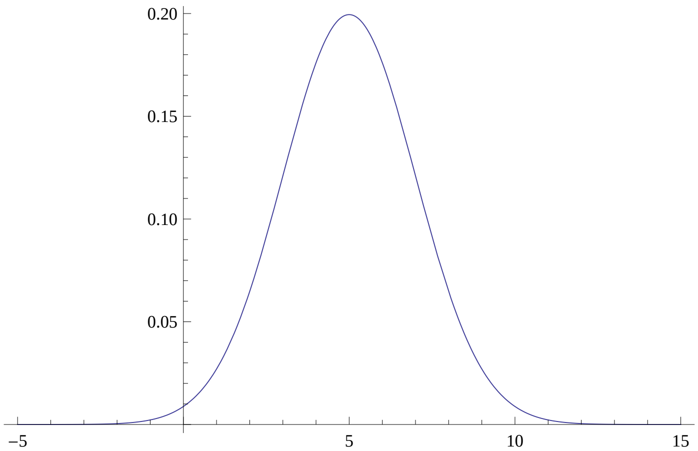
```
StandardDeviation[NormalDistribution[μ, σ]]
```

$\sigma$

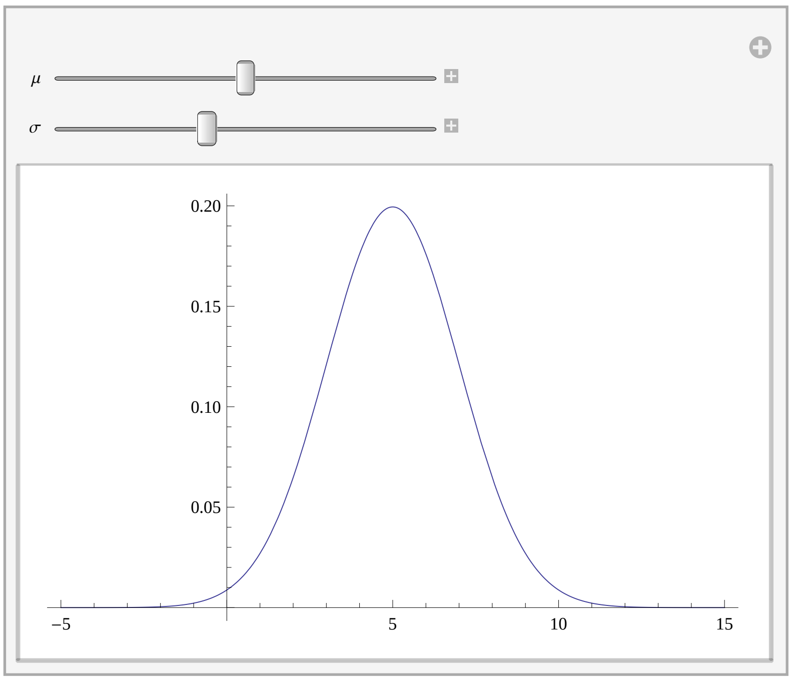
```
StandardDeviation[NormalDistribution[5, 2]]
```

2

```
Plot[PDF[NormalDistribution[5, 2], x], {x, -5, 15}]
```



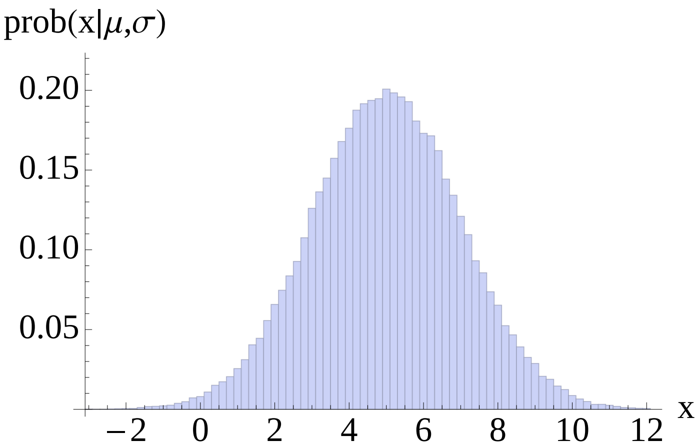

```
Manipulate[Plot[PDF[NormalDistribution[μ, σ], x], {x, -5, 15}, PlotRange → All],
 {{μ, 5}, 0, 10}, {{σ, 2}, 0.1, 5}]
```




Zufallszahlen aus der Normalverteilung ziehen und Histogram plotten:

```
In[96]:= r = RandomVariate[NormalDistribution[5, 2], 100000];
```

In[97]:=
```
Histogram[r, {-3.1, 12.1, 0.2}, "ProbabilityDensity",
 LabelStyle → 18, AxesLabel → {"x", "prob(x|μ,σ)"}]
```

Out[97]=



# Die Studentsche t-Verteilung

```
PDF[StudentTDistribution[μ, β, ν], x]
```

$$\frac{\left(\frac{\nu}{\frac{(x-\mu)^2}{\beta^2}+\nu}\right)^{\frac{1+\nu}{2}}}{\beta\sqrt{\nu}\ \mathrm{Beta}\left[\frac{\nu}{2},\frac{1}{2}\right]}$$

```
PDF[StudentTDistribution[4, 2, 3], x]
```

$$\frac{3\sqrt{3}}{\pi\left(3+\frac{1}{4}(-4+x)^2\right)^2}$$

```
Mean[StudentTDistribution[μ, β, ν]]
```

$$\begin{cases} \mu & \nu > 1 \\ \text{Indeterminate} & \text{True} \end{cases}$$

```
Mean[StudentTDistribution[4, 2, 3]]
```

4

```
Variance[StudentTDistribution[μ, β, ν]]
```

$$\begin{cases} \frac{\beta^2 \nu}{-2+\nu} & \nu > 2 \\ \text{Indeterminate} & \text{True} \end{cases}$$

```
Variance[StudentTDistribution[4, 2, 3]]
```

12

```
CentralMoment[StudentTDistribution[μ, β, ν], 2]
```

$$\begin{cases} \frac{\beta^2 \nu}{2\left(-1+\frac{\nu}{2}\right)} & 2<\nu \\ \text{Indeterminate} & \text{True} \end{cases}$$

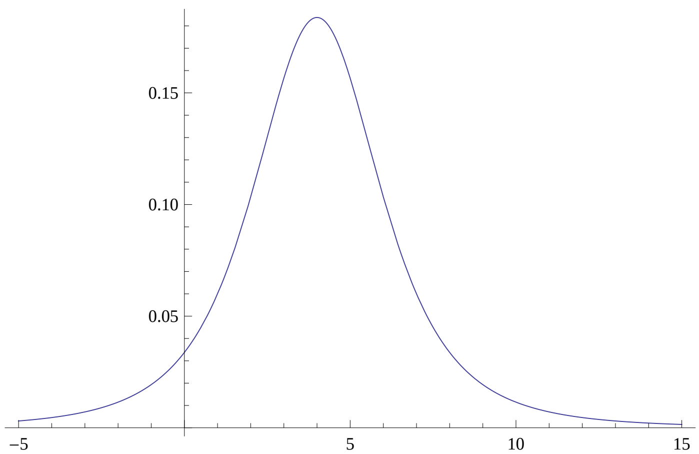
```
StandardDeviation[StudentTDistribution[μ, β, ν]]
```

$$\begin{cases} \beta \sqrt{\frac{\nu}{-2+\nu}} & \nu>2 \\ \text{Indeterminate} & \text{True} \end{cases}$$

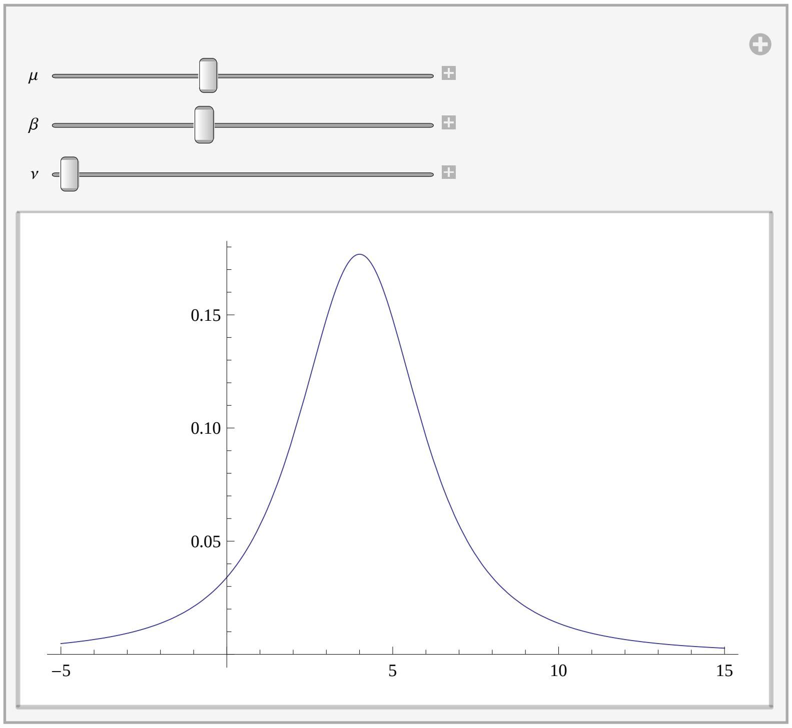
```
StandardDeviation[StudentTDistribution[4, 2, 3]]
```

$2\sqrt{3}$

```
Plot[PDF[StudentTDistribution[4, 2, 3], x], {x, -5, 15}]
```



```
Manipulate[Plot[PDF[StudentTDistribution[μ, β, ν], x], {x, -5, 15}, PlotRange → All],
 {{μ, 4}, 0, 10}, {{β, 2}, 0.1, 5}, {{ν, 2}, 2, 100}]
```

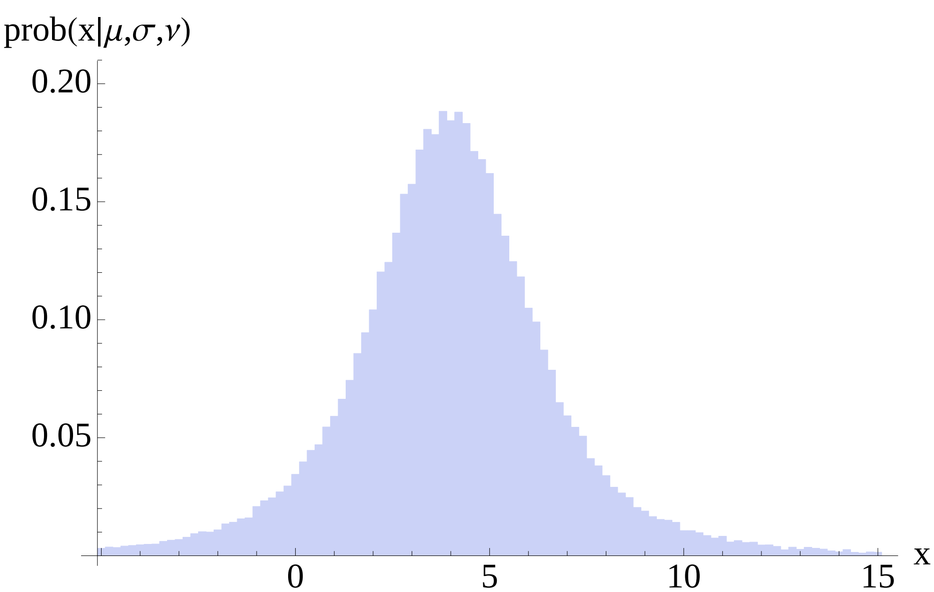



Zufallszahlen aus der Studentschen t-Verteilung ziehen und Histogram plotten:

```
In[93]:= r = RandomVariate[StudentTDistribution[4, 2, 3], 100 000];

In[94]:= Histogram[r, {-5.1, 15.1, 0.2}, "ProbabilityDensity",
          LabelStyle → 18, AxesLabel → {"x", "prob(x|μ,σ,ν)"}]
```

Out[94]=



# Die inverse Gammaverteilung

```
PDF[InverseGammaDistribution[ν, β], x]
```

$$\begin{cases} \frac{e^{-\frac{\beta}{x}} \left(\frac{\beta}{x}\right)^{\nu}}{x\,\text{Gamma}[\nu]} & x > 0 \\ 0 & \text{True} \end{cases}$$

```
PDF[InverseGammaDistribution[4, 1], x]
```

$$\begin{cases} \frac{e^{-1/x}}{6\,x^5} & x > 0 \\ 0 & \text{True} \end{cases}$$

```
Mean[InverseGammaDistribution[ν, β]]
```

$$\begin{cases} \frac{\beta}{-1+\nu} & \nu > 1 \\ \text{Indeterminate} & \text{True} \end{cases}$$

```
Mean[InverseGammaDistribution[4, 1]]
```

$$\frac{1}{3}$$

```
Variance[InverseGammaDistribution[ν, β]]
```

$$\begin{cases} \frac{\beta^2}{(-2+\nu)\,(-1+\nu)^2} & \nu > 2 \\ \text{Indeterminate} & \text{True} \end{cases}$$

```
Variance[InverseGammaDistribution[4, 1]]
```

$$\frac{1}{18}$$

```
CentralMoment[InverseGammaDistribution[ν, β], 2]
```

$$\begin{cases} \frac{\beta^2 \left(-1+\frac{1-\nu}{2-\nu}\right)}{(1-\nu)^2} & 2 < \nu \\ \text{Indeterminate} & \text{True} \end{cases}$$

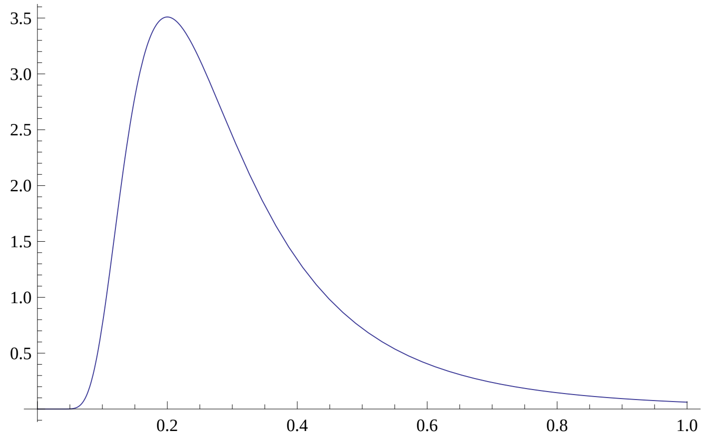
```
StandardDeviation[InverseGammaDistribution[ν, β]]
```

$$\begin{cases} \frac{\beta}{\sqrt{-2+\nu}\,(-1+\nu)} & \nu > 2 \\ \text{Indeterminate} & \text{True} \end{cases}$$

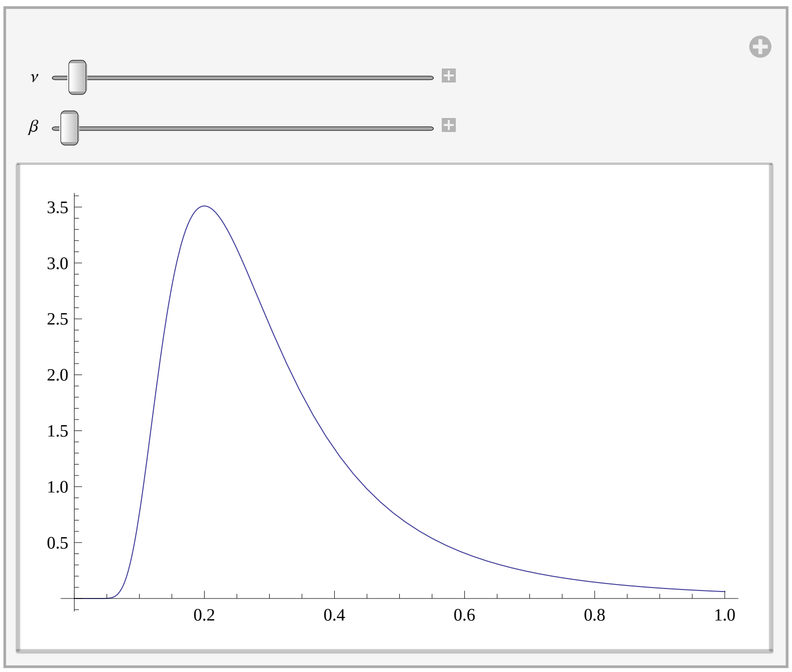
```
StandardDeviation[InverseGammaDistribution[4, 1]]
```

$$\frac{1}{3\sqrt{2}}$$

```
Plot[PDF[InverseGammaDistribution[4, 1], x], {x, 0, 1}, PlotRange → All]
```




```
Manipulate[Plot[PDF[InverseGammaDistribution[ν, β], x], {x, 0, 1}, PlotRange → All],
 {{ν, 4}, 2, 100}, {β, 1, 10}]
```




Zufallszahlen aus der inversen Gammaverteilung ziehen und Histogram plotten:

```
In[101]:= r = RandomVariate[InverseGammaDistribution[4, 1], 100 000];
```

In[102]:=
```
Histogram[r, {-0.01, 1.01, 0.02}, "ProbabilityDensity",
 LabelStyle → 18, AxesLabel → {"x", "prob(x|ν,β)"}]
```

Out[102]=