Example with data

In[1]:=
```
Needs["ErrorBarPlots`"]
```

Reading the data

These are the known errors of the data:

In[2]:=
```
σx = 0.5; σy = 0.8;
```

In[3]:=
```
data = Import["/Users/ihn/Documents/Teaching/VP-Leitung/DataAnalysis/2014/9.
    Vorlesung/errorxydata.csv", "CSV"];
```

In[4]:=
```
Grid[Prepend[data, {Style["x", Bold], Style["y", Bold]}],
 Background → {None, {Gray, {LightGray, White}}},
 Dividers → {Black, {Black, Black}}, Frame → True]
```

Out[4]=

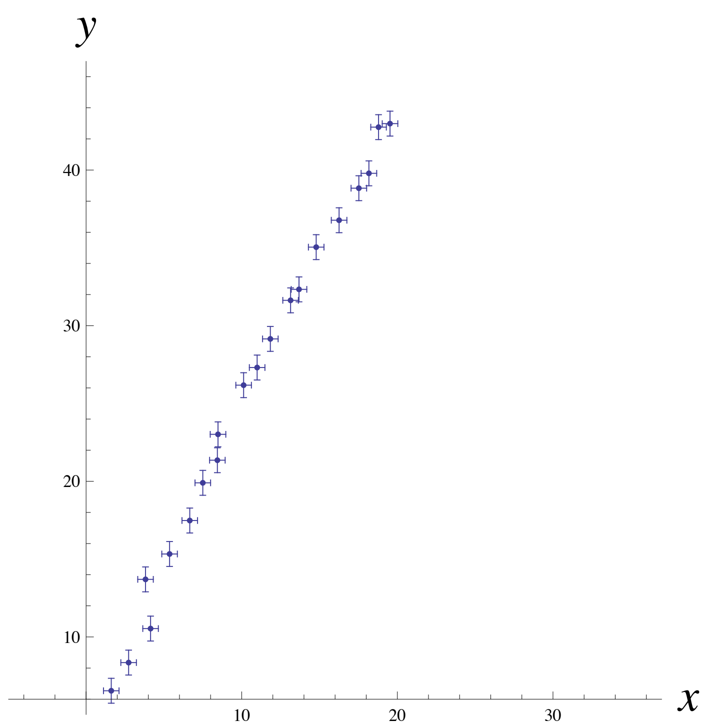
x	**y**
1.61744	6.5418
2.73276	8.35164
4.1434	10.5369
3.81891	13.6996
5.36452	15.3299
6.66155	17.48
7.49979	19.9025
8.43607	21.3554
8.47701	23.0184
10.1221	26.1769
10.9913	27.3105
11.8399	29.151
13.1402	31.6282
13.681	32.3337
14.7855	35.0458
16.2567	36.7751
17.526	38.8336
18.1723	39.7895
18.7922	42.7563
19.529	42.983

In[5]:=
```
nn = Length[data];
```

Plotting the data

In[6]:=
```
datae = Table[{data[[n]], ErrorBar[σx, σy]}, {n, 1, nn}];
g1 = ErrorListPlot[datae, PlotRange → {{-5, 2 * nn + 7 - 10}, {5, 2 * nn + 7}},
  AspectRatio → 1, AxesLabel → {Style["x", 24], Style["y", 24]}]
```

Out[7]=



Statistics of the data

In[8]:=
```
xav = Mean[data[[All, 1]]]
yav = Mean[data[[All, 2]]]
Varx = CentralMoment[data[[All, 1]], 2]
Vary = CentralMoment[data[[All, 2]], 2]
rho  = CentralMoment[data, {1, 1}] / Sqrt[Varx Vary]
```

Out[8]= 10.6794

Out[9]= 25.95

Out[10]= 30.5688

Out[11]= 126.999

Out[12]= 0.995738

-log-posterior for a and R

In[13]:=
```
LL[a_, R_, σx_, σy_, ρ_, n_] :=
```
$$\text{Log}[R\,a] + \frac{n}{2}\,\text{Log}\left[\sigma x^2\,R^2\,a + \sigma y^2\,R^2 \big/ a + \sigma x^2\,\sigma y^2\right] + \frac{n}{2}\,\frac{R^2\,a^2 - 2\,R^2\,\rho\,a + \left(\sigma x^2 + \sigma y^2\right)\,a + R^2}{\sigma x^2\,R^2\,a^2 + \sigma x^2\,\sigma y^2\,a + R^2\,\sigma y^2}$$

Minimizing the posterior for a and R

In[14]:=
```
res1 = FindMinimum[LL[aa, R, σx / Sqrt[Varx], σy / Sqrt[Vary], rho, nn], {{aa, 1}, {R, 1}}]
```
Out[14]= {-26.8478, {aa → 1.00043, R → 0.973192}}

In[15]:=
```
LLmin = res1[[1]];
aest = aa /. res1[[2]]
Resti = R /. res1[[2]]
```
Out[16]= 1.00043

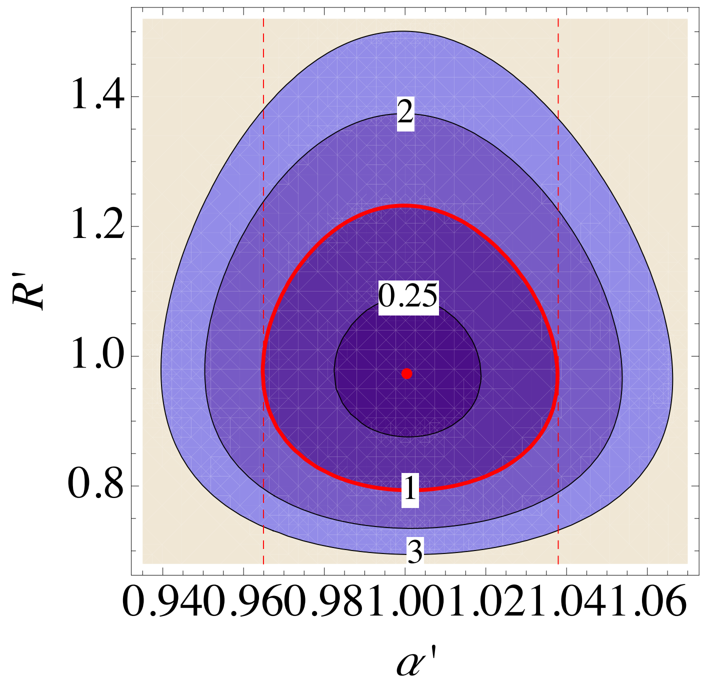
Out[17]= 0.973192

Plotting the -log-posterior for a and R

We determine the uncertainties of a graphically from this plot:

In[18]:=
```
Show[ContourPlot[LL[aa, R, σx / √Varx , σy / √Vary , rho, nn] - LLmin,
   {aa, 0.935, 1.07}, {R, 0.68, 1.52}, Contours → {0.25, 1, 2, 3},
   FrameLabel → {Style["α'", 24], Style["R'", 24]}, FrameTicksStyle → 24,
   ContourLabels → (Text[Style[#3, 18], {#1, #2}, Background → White] &),
   ContourStyle → {Black, {Red, AbsoluteThickness[2]}, Black, Black}],
  Graphics[{{Red, Disk[{aest, Resti}, {0.01 (1.07 - 0.935), 0.01 (1.52 - 0.68)}]},
    {Red, Dashed, Line[{{aest + 0.0375, 0.68}, {aest + 0.0375, 1.52}}],
     Line[{{aest - 0.0355, 0.68}, {aest - 0.0355, 1.52}}]}}]]
```

Out[18]=



In[19]:=
```
Δap = 0.0375;
Δam = 0.0355;
```

Estimate of the slope and the intercept parameters α and β

Slope parameter:

In[21]:=
```
α = aest √Vary / √Varx
```

Out[21]= 2.03914

In[22]:=
```
Δαp = Δap √Vary / √Varx
Δαm = Δam √Vary / √Varx
```

Out[22]= 0.0764349

Out[23]= 0.0723584

Intercept parameter:

In[24]:= $\beta = \text{yav} - \text{xav}\,\alpha$

Out[24]= 4.17322

In[25]:= $\Delta\beta = \sqrt{\frac{\sigma y^2 + \sigma x^2\,\alpha}{\text{nn}} + \text{xav}^2\,\Delta\alpha p\,\Delta\alpha m}$

Out[25]= 0.829616

Plotting the result of the fit

In[26]:=
```
Show[g1, Plot[{α (x - xav) + yav, (α + Δαp) (x - xav) + yav, (α - Δαm) (x - xav) + yav},
  {x, -5, 2 * nn + 7 - 10},
  PlotStyle → {{Red, AbsoluteThickness[2]}, {Red, Dashed}, {Red, Dashed}}]]
```

Out[26]=